Absorption of electromagnetic waves in two-dimensional systems under a magnetic field and a periodic potential

1997 J. Phys.: Condens. Matter 9 L641

(http://iopscience.iop.org/0953-8984/9/49/002)

J. Phys.: Condens. Matter **9** (1997) L641–L646. Printed in the UK PII: S0953-8984(97)87917-9

LETTER TO THE EDITOR

Absorption of electromagnetic waves in two-dimensional systems under a magnetic field and a periodic potential

Arne Brataas†‖, C Zhang‡ and K A Chao†§

† Department of Physics, Norwegian University of Science and Technology, N-7034 Trondheim, Norway

‡ Department of Physics, University of Wollongong, New South Wales 2522, Australia

§ Department of Theoretical Physics, Lund University, S-223 26 Lund, Sweden

Received 25 September 1997

Abstract. The interaction of an electromagnetic wave with a two-dimensional electron gas subjected to a magnetic field and a weak periodic potential is investigated. The electron–impurity interaction is included to the lowest order. The periodic modulation of the Landau states has a profound effect on the optical conductivity. Simultaneous excitation of an electron–hole pair contributes significantly to the absorption around and below the cyclotron frequency. This is in contrast to a uniform electron gas under a perpendicular magnetic field where electron–hole pair excitations are absent.

A two-dimensional electronic system subjected to a perpendicular magnetic field and a periodic modulation potential [1–12] represents an interesting and challenging problem in physics, mathematics and computer simulation techniques. Since the discovery of the Weiss oscillations in a weakly modulated system [1], research in this field has been rapidly growing. Recent progress includes the experimental realization of the internal structures of the Landau bands [13, 14], and quantum chaotic dynamics [15]. The effect of a unidirectional periodic potential on the static properties can be summarized as: (i) the sharp Landau levels are broadened so that the width and height of the density of states (DOS) are oscillatory with magnetic field and level index; (ii) the DOS exhibits inverse-square-root singularities at the band edges; (iii) there is an additional contribution to the conductivity due to the band dispersion; (iv) the magnetoresistivity exhibits commensurability oscillations (or Weiss oscillations). However, to date, the dynamical properties of this subtle system are still far from clear. In a recent work we performed a calculation of the dynamical density response function [16, 17]. It was revealed that the effect of the periodic potential on the dynamical properties of the system is more complicated than that on the static properties: (i) it introduces an additional channel in the density response which is due to the electron–hole pair excitations; (ii) the pair excitation exhibits multiple singularities; (iii) for partially filled Landau bands, the light scattering cross section has additional peaks, which at half-filling is at the cyclotron frequency and its high harmonics.

The primary effect of a weak periodic potential is to lift the degeneracy of the original Landau levels. The energy becomes dependent on the centre coordinate of the cyclotron orbit, $x_o = q_y l^2$, where $l^2 = c\bar{h}/(eB)$ is the magnetic length. In this letter, we demonstrate that an additional weak periodic potential applied to a uniform 2DEG under a

‖ Present address: Delft University of Technology, Faculty of Applied Physics, Lorentzweg 1, 2628 CJ Delft, The Netherlands.

0953-8984/97/490641+06$19.50

magnetic field will yield the following important results for the electromagnetic absorption: (i) the centre coordinate dependent energy dispersion has introduced the electron–hole pair excitation. This pair excitation is the dominant absorption mechanism for $\omega < \omega_c$, where $\omega_c = eB/(cm^*)$ is the cyclotron frequency; (ii) the absorption due to plasmon excitation is no longer singular because the spectral weight is now distributed continuously over a finite interval of wave vectors.

Let us consider a 2DEG in the x–y plane with a perpendicular magnetic field B, subjected to a periodic modulation in the x-direction. The Hamiltonian is

$$H = \sum_i \frac{p_i^2}{2m^*} + \sum_{i<j} \frac{e^2/\kappa}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} + \sum_i V_0 \cos(K x_i) + \sum_{\boldsymbol{q},i,j} U_q \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{R}_j)} \tag{1}$$

where $\boldsymbol{r}_i = (x_i, y_i)$ is the electron position, $\boldsymbol{p}_i = (-\mathrm{i}\hbar\nabla_i - \mathrm{e}\boldsymbol{A}_i/c)$ is the electron momentum, $\boldsymbol{A}_i$ is the vector potential of the static magnetic field, $\boldsymbol{R}_j$ is the impurity coordinate, and V_0 is the strength of the modulation with period $a = 2\pi/K$. We use the electron–impurity interaction $U_q = 2\pi e^2(\exp(-q\alpha))/\kappa q$, where α is the typical distance of the doping impurities from the 2DEG in a semiconductor heterostructure. The mass of the impurity is treated as infinite. The interaction of an EM wave with the electrons is $H_\gamma = -e\boldsymbol{E}_\gamma \cdot \sum_i \boldsymbol{p}_i/(m^*\mathrm{i}\omega)$, where $\boldsymbol{E}_\gamma$ is the transverse electric field and ω is the frequency of the EM wave. We limit our discussion to low temperatures and set $T = 0$. All results may easily be generalized to finite temperatures.

The energy loss rate of the EM field can be written using the Fermi golden rule as

$$R(\omega) = \frac{2\pi}{\hbar}\frac{e^2}{m^2}\frac{1}{\omega^2}(E_F - E_0 - \hbar\omega)\left|\sum_{\boldsymbol{q}}\left(\frac{q_- E_+}{\omega + \omega_c} + \frac{q_+ E_-}{\omega - \omega_c}\right)U_q \right.$$
$$\left. \times \langle F|\sum_{i,j}\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{R}_j)}|0\rangle + \frac{\mathrm{i}KV_o}{\sqrt{2}}\left(\frac{E_+}{\omega + \omega_c} + \frac{E_-}{\omega - \omega_c}\right)\langle F|\sum_i \sin(Kx_i)|0\rangle\right|^2 \tag{2}$$

where $|F\rangle$ and $|0\rangle$ are, respectively, the excited and ground state of the many-electron system, $q_\pm = (q_x \pm \mathrm{i}q_y)/\sqrt{2}$ and $E_\pm = (E_x \pm \mathrm{i}E_y)/\sqrt{2}$. This expression is exact to second order in the impurity potential U_q and the modulation potential V_0, including all electron–electron interactions and is valid for any strength of the magnetic field. The electron scattering matrix contains two terms. The first term, which is proportional to $n_I q U_q$, is due to electron scattering off the impurity potential, and the second term, which is proportional to KV_o, is due to the electron scattering off the modulation potential. Here n_I is the density of impurities. In typical experimental situations, $V_o \sim$0.5–2 meV, $K \sim$2–4$\times 10^5$ cm^{-1} and $n_I =$2–5$\times 10^{11}$ cm^2. Therefore the ratio $n_I q U_q/KV_o \gg 1$, and we will neglect the electron scattering from the weak periodic potential. Furthermore, since we are seeking a result which is in the lowest order in electron–impurity interaction, i.e. in the order $|U_q|^2$, the many-body states $|F\rangle, |I\rangle$ can be regarded as only having electron coordinates. In this case $\sum_j \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{R}_j}$ can be taken out of the matrix element and we can average over the random impurity ensemble $\left(\sum_{j,j'}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{R}_j}\mathrm{e}^{\mathrm{i}\boldsymbol{q}'\cdot\boldsymbol{R}_{j'}}\right)/A = n_I\delta_{\boldsymbol{q},\boldsymbol{q}'}$, where A is the area of the two-dimensional system.

We can relate the energy loss rate to the conductivity by $\hbar\omega R(\omega) = 2A\mathrm{Re}[\boldsymbol{j}(\omega)\cdot\boldsymbol{E}^*(\omega)]$. The current can be written as $\boldsymbol{j} = \sigma\cdot\boldsymbol{E}$ where σ is the conductivity, and we thus obtain the real part of the diagonal element of the conductivity tensor

$$\mathrm{Re}[\sigma_{xx}] = \eta\sum_{\boldsymbol{q}}(q_x^2\omega^2 + q_y^2\omega_c^2)\frac{|U_q|^2}{V_q}\left[-\mathrm{Im}\frac{1}{\epsilon(\boldsymbol{q},\omega)}\right] \tag{3}$$

$$\mathrm{Re}[\sigma_{yy}] = \eta \sum_{\boldsymbol{q}} (q_x^2\omega_c^2 + q_y^2\omega^2)\frac{|U_q|^2}{V_q}\left[-\mathrm{Im}\frac{1}{\epsilon(\boldsymbol{q},\omega)}\right] \tag{4}$$

where $V_q = 2\pi e^2/\kappa q$ and the prefactor is $\eta = n_I(e/m^*)^2(\omega(\omega^2-\omega_c^2)^2)^{-1}$. Due to the unidirectional modulation potential, the conductivity is now, in general, anisotropic. However, we see from (3) and (4) that close to the cyclotron frequency, $\omega \sim \omega_c$, the anisotropy in the conductivity is small, even if the dielectric function is strongly anisotropic. The anisotropy increases when the frequency deviates from the cyclotron frequency.

For the absorption in the system, we now need a model for the dielectric function in a 2DEG with a perpendicular magnetic field and a periodic modulation potential. To this end, we will use the results obtained within the random-phase approximation [16, 17, 18]. To simplify the discussion, we will consider the case of integer filling of the Landau levels. The single-particle energy, to first order in the modulation potential, is given as $E_n(x_0) = \bar{h}\omega_c(n+1/2) + U_n \cos Kx_0$, where $U_n = V_0 L_n(\mathcal{H})\exp(-\mathcal{H}/2)$, $\mathcal{H} = (Kl)^2/2$ and $L_n(\mathcal{H})$ is a Laguerre polynomial. This is a good approximation if the cyclotron resonance and the Fermi energy are not too small compared to the modulation potential. The imaginary part of the dielectric function is [18, 16]

$$\mathrm{Im}[\epsilon(\boldsymbol{q},\omega)] = \frac{2\bar{h}\omega_c}{qa_B^*}\sum_{m'=0}^{n_F}\sum_{m=x}^{\infty} C_{m+m',m'}\Big[\theta\left(\Delta U_{mm'}^2 - (\Delta E_m^-)^2\right)/\sqrt{\Delta U_{mm'}^2 - (\Delta E_m^-)^2}$$
$$-\theta\left(\Delta U_{mm'}^2 - (\Delta E_m^+)^2\right)/\sqrt{\Delta U_{mm'}^2 - (\Delta E_m^+)^2}\Big]. \tag{5}$$

In this equation $n_F + 1$ is the number of occupied Landau bands, $x = n_F + 1 - m'$ is the lower limit of the second summation, and the transition matrix for $n' < n$ is

$$C_{n,n'} = \frac{n'!}{n!}X^{n-n'}\mathrm{e}^{-X}[L_{n'}^{n-n'}(X)]^2 \tag{6}$$

where $X = (ql)^2/2$ and $L_n^m(X)$ is an associated Laguerre polynomial. The real part of the dielectric function is [16, 18]

$$\mathrm{Re}[\epsilon(\boldsymbol{q},\omega)] = 1 + \frac{2\bar{h}\omega_c}{qa_B^*}\sum_{m'=0}^{n_F}\sum_{m=x}^{\infty} C_{m+m',m'}\Big[\theta((\Delta E_m^-)^2 - \Delta U_{mm'}^2)/\Delta E_m^-$$
$$\times\sqrt{1-(\Delta U_{mm'}/\Delta E_m^-)^2} + \theta((\Delta E_m^+)^2 - \Delta U_{mm'}^2)/\Delta E_m^+\sqrt{1-(\Delta U_{mm'}/\Delta E_m^+)^2}\Big] \tag{7}$$

where $\theta(x)$ is the Heaviside function. We have introduced

$$\Delta U_{mm'}^2 = U_{m+m'}^2 - 2U_{m+m'}U_{m'}\cos Kx_0' + U_{m'}^2 \tag{8}$$

where $\Delta E_m^{\pm} = m\bar{h}\omega_c \pm \bar{h}\omega$, $x_0' = q\sin(\vartheta)l^2$ and ϑ is the angle between the wave vector $\boldsymbol{q}$ and the x-axis. For the singularities in $\mathrm{Im}[\epsilon(\boldsymbol{q},\omega)]$ we see that for positive frequencies they occur at $\Delta E_m^- = \pm\Delta U_{mm'}$ if subband m' is occupied and subband $m+m'$ is empty [16]. Now, since the conductivity given by (3) and (4) in principle contains contributions from all (q,ϑ) due to the scattering off the impurities, the singularities will occur in the interval $||U_{m+m'}| - |U_{m'}|| \leqslant |\Delta E_m^-| \leqslant ||U_{m+m'}| + |U_{m'}||$ and will be smoothed out.

The EM absorption of the system consists of contributions from both single-particle excitations and plasmon excitations. We can separate these terms as $-\mathrm{Im}[1/\epsilon] = \mathrm{P}\{\mathrm{Im}[\epsilon]/(\mathrm{Im}[\epsilon])^2 + (\mathrm{Re}[\epsilon])^2\} + \pi\delta(\epsilon)$, where P denotes the principal part. From this relation, it is well known that in the absence of a periodic modulation potential, the EM response of a system with sharp Landau levels does not contain a contribution due to the particle–hole pair excitation. The contribution to the EM response due to plasmon excitation in such a system usually has sharp peaks due to the fact that for a given value of the photon

wave vector, the plasmon energy is independent on the angle of the wave vector. The application of a modulation potential has broadened the originally sharp Landau level. The broadening depends on both the magnetic field and the Landau level index. The centre coordinate dependent energy dispersion has two important effects on the EM response of the system: (i) the particle–hole pair excitation channel has now been opened, and thus a finite absorption occurs for frequencies around ω_c and each of its harmonics ($n\omega_c$, where $n = 1, 2, 3, \ldots$). Therefore in a modulated 2DEG, the EM absorption is finite even for frequencies less than ω_c. (ii) the spectral weight in plasmon excitation is no longer only dependent on the magnitude of the photon wave vector. It now also depends on the direction of the photon wave vector. The spread of the spectral weight along different directions (specified by the angular variable ϑ) removes the singular behaviour (or smears out the sharp peaks) in the absorption coefficient. The contribution from collective excitations to the conductivity component $\mathrm{Re}[\sigma_{xx}]$ is then given as

$$\mathrm{Re}[\sigma^c_{xx}] = \frac{1}{2}\eta(\omega)\int_0^{2\pi}\frac{\mathrm{d}\vartheta}{2\pi}\sum_{q^*}\left[q(q_x^2\omega^2 + q_y^2\omega_c^2)\frac{|U_q|^2}{V_q}\left|\frac{\partial\epsilon(q,\vartheta,\omega)}{\partial q}\right|^{-1}\right]_{q=q^*} \tag{9}$$

where the q^*s are solutions of $\epsilon(q^*, \vartheta, \omega) = 0$. All singularities will thus be smoothed out in the angular integral. We illustrate this behaviour in figure 1, where we show the dispersion $\omega(q)$ of the plasmon as a function of the wave vector q for two values of the angle ($\vartheta = 0, \pi/2$). The system parameters used in the calculation are to be described below. The areas below the dashed lines are the regimes of single-particle excitations, where the collective excitations (plasmon) cannot exist. For $\vartheta = 0$ ($q_y = 0$, $q_x = q$), this regime is independent of q, but for $\vartheta = \pi/2$ ($q_y = q$, $q_x = 0$), this regime increases as a function of q. The angular dependence of the plasmon energy is stronger for small frequency differences, $\omega(q) - \omega_c$, as expected from the expression for the real part of the

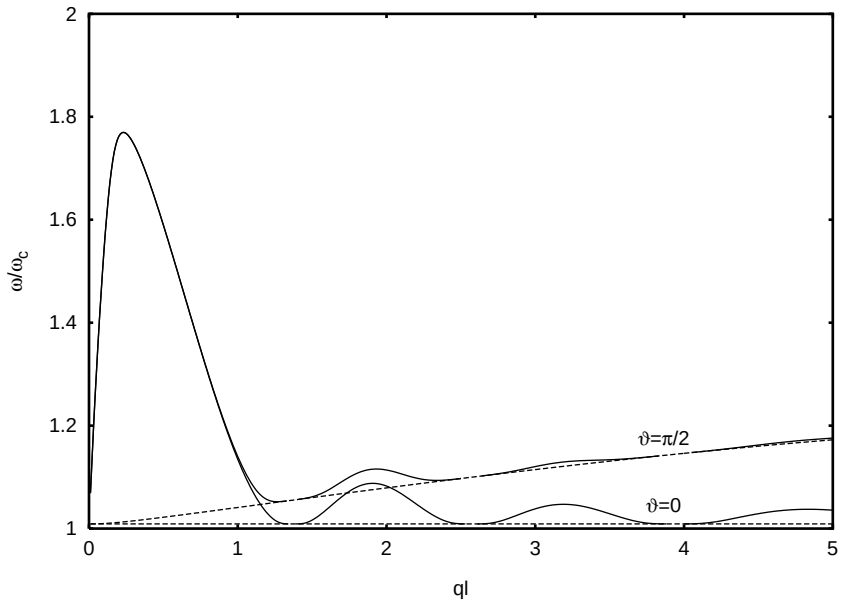

Figure 1. Normalized plasmon frequency $\omega(q)/\omega_c$ (solid curves) as a function of normalized wave vector ql for a fixed angle $\vartheta = 0$ and $\vartheta = \pi/2$. The areas below the dashed lines are the regimes of single-particle excitations. The system parameters are described in the text.

dielectric function (7), since the angular dependent terms in the denominator are then more important. The highest peak in the dispersion has a rather weak angular dependence, since the dispersive part of energies in the real part of the dielectric function is less pronounced for larger energy differences, $\omega(q) - \omega_c$. This means that the peak in the conductivity within the RPA around this frequency will remain sharp after the integration in (9), while the other peaks will be more smoothed out. However, with a slight increase of V_0, the maximum plasmon frequency may overlap with the particle–hole region around $2\omega_c$, and so be damped out.

We have used the following GaAs parameters in our numerical calculation $m^* = 0.067m$, $\kappa = 13$ (m is the electron mass). This gives an effective Bohr radius of $a_B^* = 103$ Å. The magnetic field is $B = 2$ T, thus giving a Landau energy $\hbar\omega_c = 3.5$ meV. With a 2D electron density of 3.9×10^{11} cm^{-2}, the four lowest Landau levels are filled. The modulation potential is $V_0 = 0.5$ meV, with a period of $a = 3000$ Å, and the typical distance from the impurities to the 2DEG is $\alpha = 100$ Å.

We consider the 'resistivity', $\mathrm{Re}[\rho_{xx}] = \mathrm{Re}[\sigma_{yy}]/|\sigma^0|^2$, $(\sigma^0 = \mathrm{i}ne^2\omega/(m(\omega^2 - \omega_c^2)))$ around ω_c. The region of single-particle excitations is determined by transitions between Landau level 3 and 4, for which $U_3 = 0.11\omega_c$ and $U_4 = 0.10\omega_c$. In principle the sum $|U_3 + U_4| = 0.21\hbar\omega_c$ gives the maximum frequency range around ω_c for single-particle excitations, and $\hbar\omega_c + |U_3 - U_4| = 1.01\hbar\omega_c$ gives the minimum frequency for which the plasmon may exist. However, as we can see from (3) and (4), the spectrum is effectively cut off if q_y is larger than $1/\alpha$, where α is the typical distance from the impurities to the 2DEG. The maximum argument for Kx_0' is thus $Kx_0' \sim 2\pi l^2/(a\alpha) \approx 0.71$, leading to a more narrow band of single-particle excitations around ω_c. We show in figure 2 the resistivity, $\mathrm{Re}[\rho_{xx}]$, as a function of frequency. The solid curve displays the total resistivity

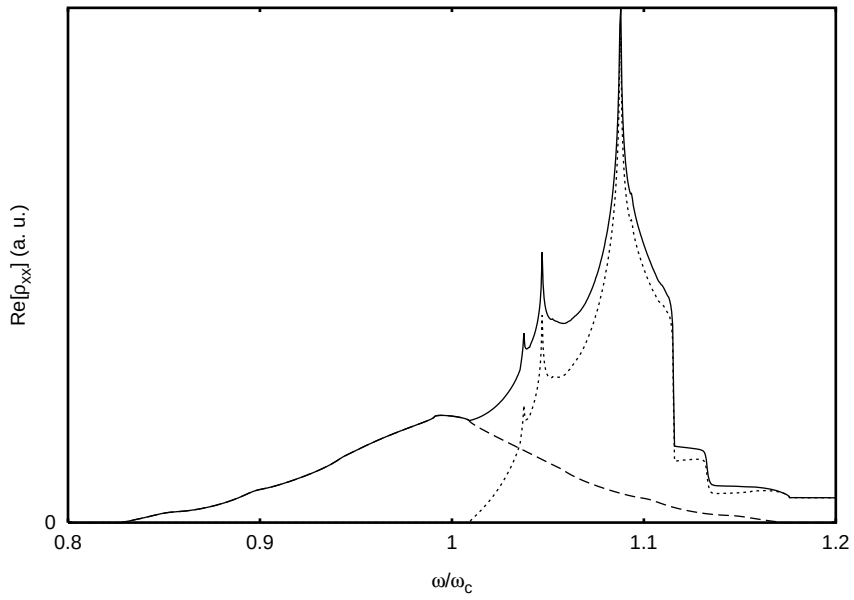

Figure 2. Resistivity, $\mathrm{Re}[\rho_{xx}]$, as a function of frequency (solid curve). The dashed curve represents the single-particle excitations, and the dotted curve the collective contributions. The system parameters are described in the text.

including contributions from both single-particle and collective excitations. For frequencies less than the cyclotron frequency, only single-particle excitations contribute (dashed curve which merges with the solid curve slightly above ω_c). At frequencies slightly larger than ω_c, $\omega = 1.01\omega_c$ (see above), the collective excitations (dotted curve) start to contribute to the conductivity and their contribution is dominant at high frequencies. The resistivities $\mathrm{Re}[\rho_{yy}]$ (not shown) and $\mathrm{Re}[\rho_{xx}]$ are almost identical even though the anisotropy of the dielectric function is large. As already mentioned this is due to the fact that we have integrated over the angular variable for both $\mathrm{Re}[\sigma_{xx}]$ and $\mathrm{Re}[\sigma_{yy}]$.

We would like to point out that in a realistic system the Landau levels are not infinitely sharp in the absence of a periodic modulation. Therefore, the particle–hole pair excitation can make a nonzero contribution to the EM absorption. Broadening of the levels comes from higher-order effects in both impurity and Coulomb interactions [19]. However, the physical origin of this commonly studied collision broadening (without modulation potential) is completely different from the physical origin of the modulation broadening discussed here. The former is known as the 'lifetime effect' and the latter is the 'dispersive effect'. The particle–hole excitations due to collision broadening and due to modulation broadening, and their respective contributions to the EM absorption are qualitatively different: (i) the former is still isotropic because the scattering centres are randomly distributed in space, while the latter is anisotropic; (ii) the EM absorption due to the former usually has a Lorentzian type spectral distribution, which is relatively sharp, while that due to the latter is definitely non-Lorentzian. The angular redistribution of the spectral weight due to plasmon excitation is also a completely new mechanism compared to the collisional damping of plasmons in unmodulated systems. The central result of this work is the novel absorption mechanism due to modulation-induced level broadening. The RPA dielectric function is employed in our work, but the qualitative physical picture should remain unchanged, if the dielectric function is calculated beyond the random phase appproximation.

References

[1] Weiss D, von Klitzing K, Ploog K and Weinmann G 1989 *Europhys. Lett.* **8** 179
[2] Winkler R W, Kotthaus J P and Ploog K 1989 *Phys. Rev. Lett.* **62** 1177
[3] Gerhardts R R, Weiss D and von Klitzing K 1989 *Phys. Rev. Lett.* **62** 1173
[4] Weiss D, Zhang C, Gerhardts R R, von Klitzing K and Weinmann G 1989 *Phys. Rev.* B **39** 13 020
[5] Pfannkuche D and Gerhardts R R 1992 *Phys. Rev.* B **46** 12 606
[6] Gerhardts R R and Zhang C 1990 *Phys. Rev. Lett.* **64** 1473
Zhang C and Gerhardts R R 1990 *Phys. Rev.* B **41** 12 850
[7] Vasilopoulos P and Peeters F M 1989 *Phys. Rev. Lett.* **63** 2120
[8] Zhang C 1990 *Phys. Rev. Lett.* **65** 2207
[9] Manolescu A and Gerhardts R R 1995 *Phys. Rev.* B **51** 1703
[10] Weiss D, Richter K, Menschig A, Bergmann R, Scheizer H, von Klitzing K and Weinmann G 1993 *Phys. Rev. Lett.* **70** 4118
[11] Hofstadter D R 1976 *Phys. Rev.* B **14** 2239
[12] Sinai Y G 1970 *Usp. Mat. Nauk.* **25** 141
[13] Schlösser T, Ensslin K, Kotthaus J P and Holland M 1996 *Semicond. Sci. Tech.* **11** 1582
[14] Schlösser T, Ensslin K, Kotthaus J P and Holland M 1996 *Europhys. Lett.* **33** 683
[15] Rotter P, Suhrke M and Rossler U 1996 *Phys. Rev.* B **54** 4452
[16] Stewart S M and Zhang C 1995 *Phys. Rev.* B **52** R17036
[17] Brataas A, Zhang C and Chao K A 1997 *Phys. Rev.* B **55** 15 423
[18] Cui H L, Fessatidis V and Horing N J M 1989 *Phys. Rev. Lett.* **63** 2598
[19] Fukuyama H and Kuramoto Y and Platzman P M 1979 *Phys. Rev.* B **19** 4980